Online appendix to "Modeling Life-Cycle Earnings Risk with Positive and Negative Shocks"

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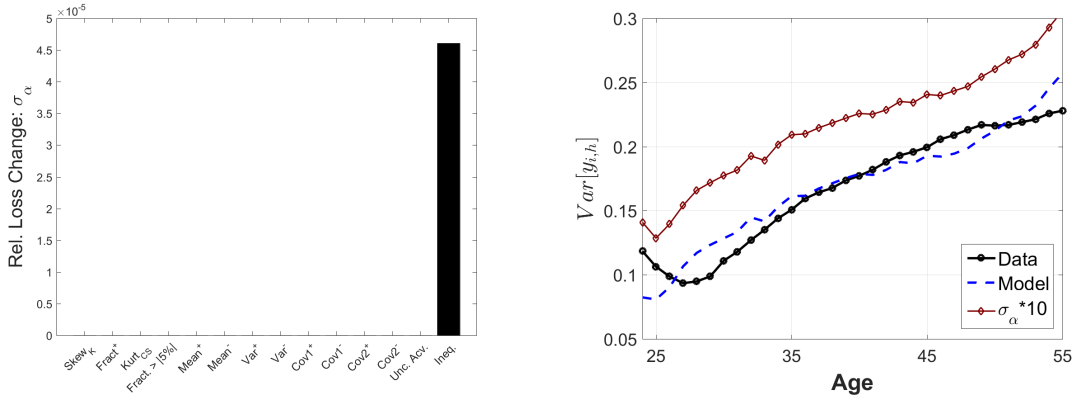
*This paper uses the Sample of Integrated Labour Market Biographies - Regional File 1975-2010, *SIAB R 7510*. The data was provided via the Cornell Restricted Access Data Center, previous authorization of the Research Data Center of the German Federal Employment Agency at the Institute for Employment Research, under the project 'Labour Income Profiles are not heterogeneous: a European test'. Felix Wellschmied gratefully acknowledges support from the Spanish Ministry of Economics through research grants *ECO2014-56384-P*, *MDM 2014-0431*, and Comunidad de Madrid *MadEco-CM (S2015/HUM-3444)* and thanks the Department of Economics at ITAM for its hospitality.

A Identification

In the following, we provide additional intuition for the identification of the parameters discussed in Section 4.4 of the paper. To this end, we perform two related simulation exercises. First, we highlight the relationship between a particular model parameter and the different data moments. To this end, we simulate a 1% change in a model parameter from its optimum holding all other parameters fixed and plot the resulting change in the age averaged model moments relative to their minimum.[1] Second, to highlight those moments providing most of the identification of a particular parameter, we plot the non-aged average change in those model moments as a response to a change in the model parameter from its optimum. In this exercise, we select changes in parameter values at discretion to make the effects best visible.

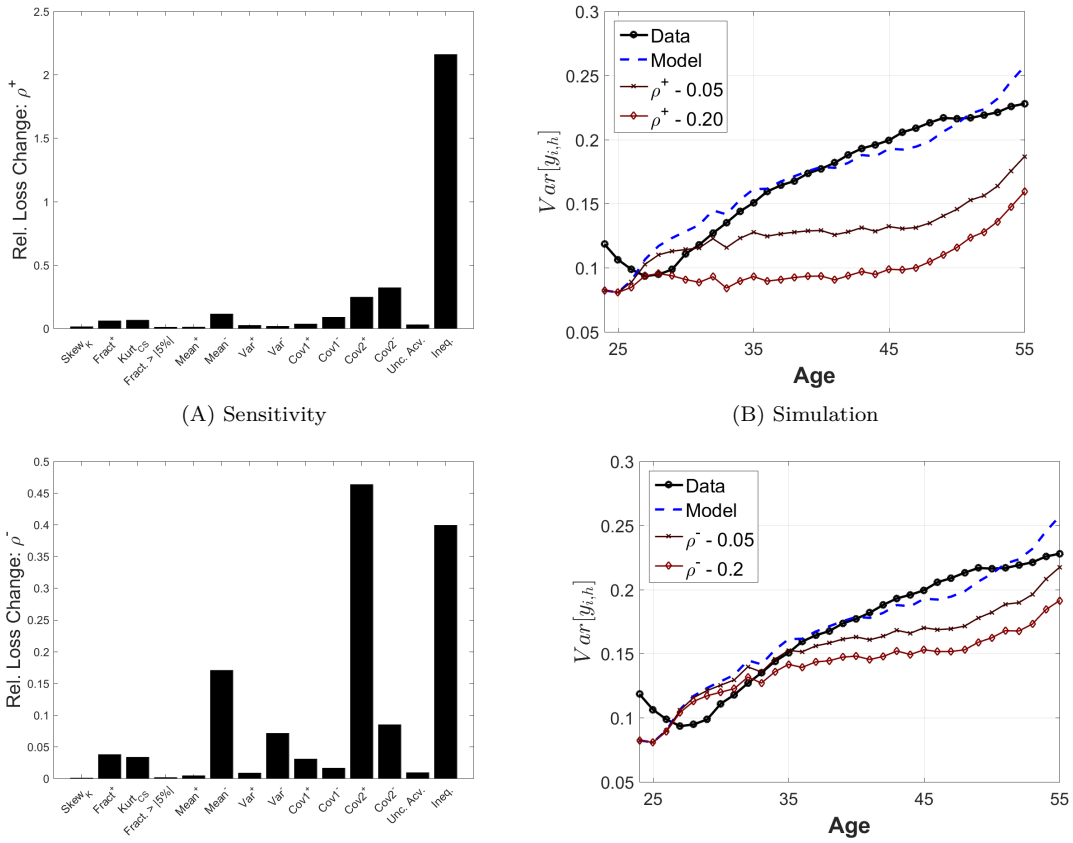


(A) Sensitivity (B) Simulation

Panel A displays the moment responses to a 1% increase in the standard deviation of permanent heterogeneity, $\hat{\sigma}_\alpha$. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panel B displays the simulated cross-sectional inequality resulting from a ten-fold increase relative to the optimum.

Figure I: Permanent initial heterogeneity

[1] All parameter changes affect the mean of log earnings and log earnings growth, and we choose to omit these responses in our graph for illustration purposes.



(A) Sensitivity

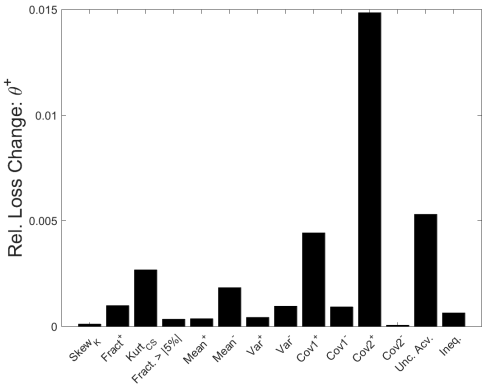
(B) Simulation

(C) Sensitivity

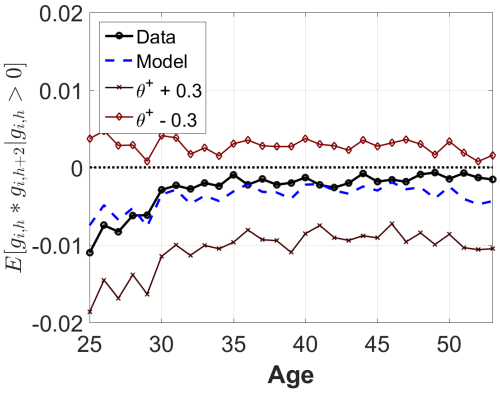
(D) Simulation

Panels A and C display the moments response to a 1% increase of the autocorrelation parameters, $\hat{\rho}^+$ and $\hat{\rho}^-$, respectively. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panels B and D display the simulated cross-sectional inequality of selected parameter values that are of moderate persistence ($\rho \approx 0.8$).

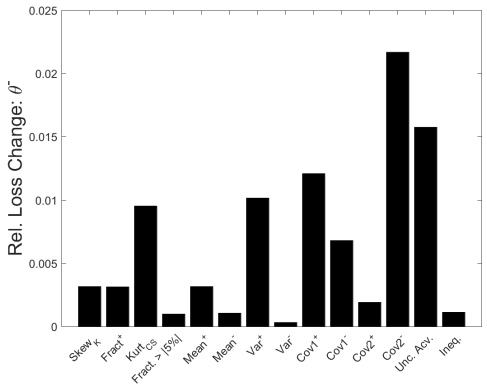
Figure II: Autocorrelation of persistent shocks



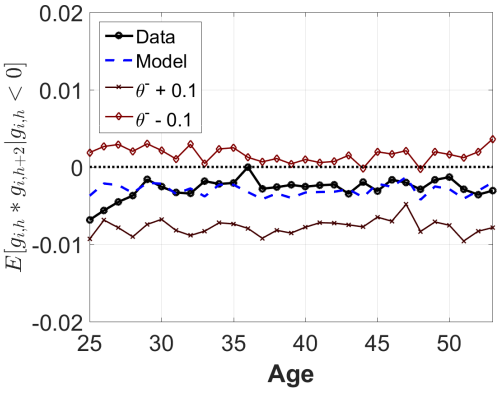
(A) Sensitivity



(B) Simulation



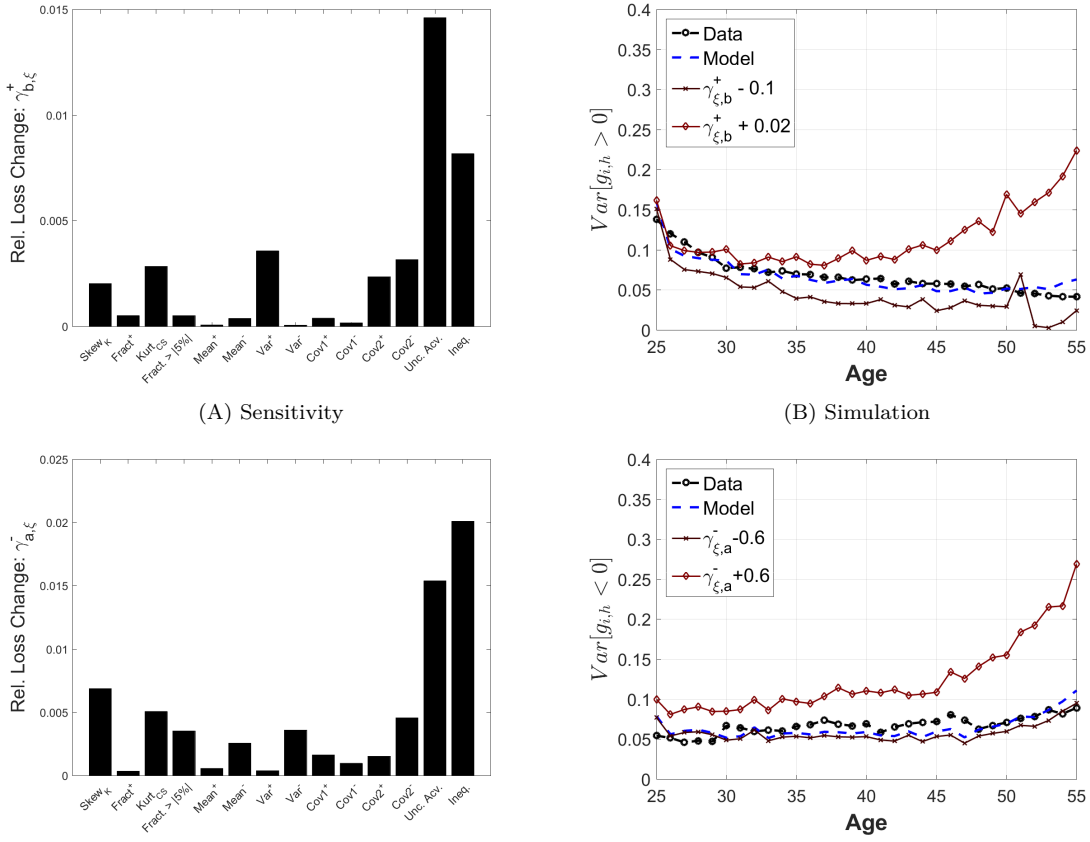
(C) Sensitivity



(D) Simulation

Panels A and C display the moments response to a 1% increase in the parameters guiding the persistence of transitory shocks, θ^+ and θ^-, respectively. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panels B and D display the simulated positive and negative first autocovariances, respectively, resulting from increasing and decreasing these parameters.

Figure III: Persistence of transitory shocks



(A) Sensitivity

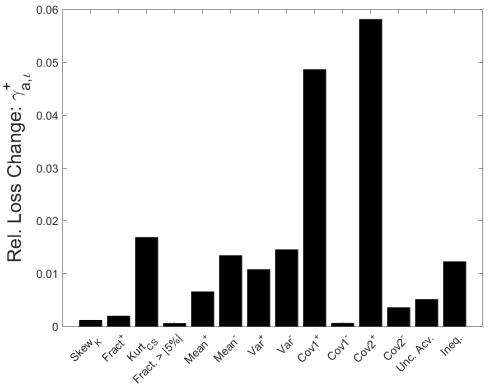
(B) Simulation

(C) Sensitivity

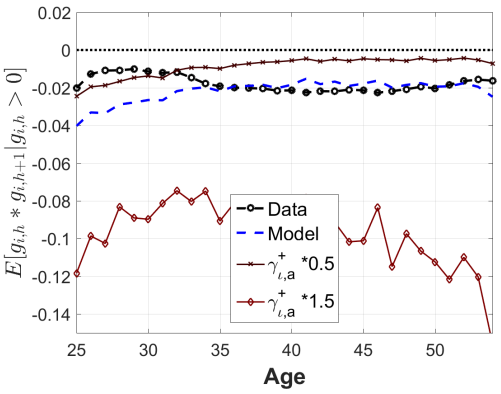
(D) Simulation

Panels A and C display the moments response to a 1% decrease in the parameters guiding the variances of persistent shocks, $\hat{\gamma}^{+}_{\xi,b}$, and a 1% increase of $\hat{\gamma}^{-}_{\xi,a}$, respectively. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panels B and D display the simulated positive and negative variance, respectively, resulting from increasing and decreasing these parameters.

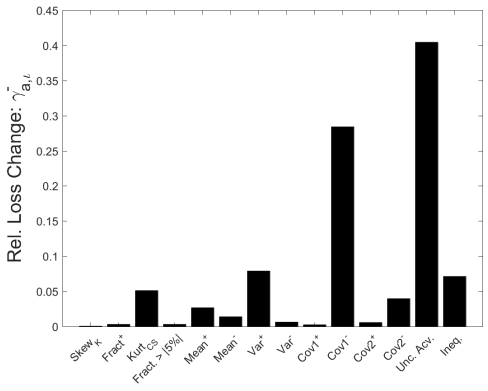
Figure IV: Variance of persistent shocks



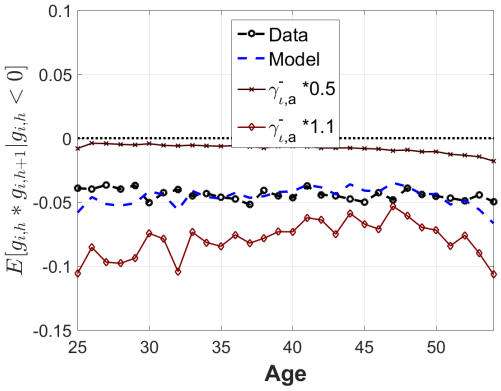
(A) Sensitivity



(B) Simulation



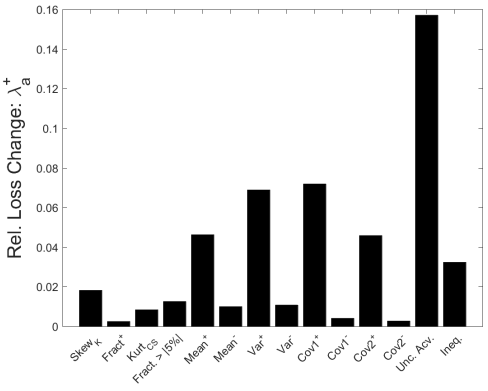
(C) Sensitivity



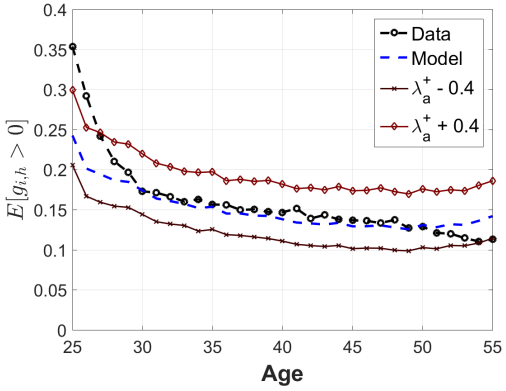
(D) Simulation

Panels A and C display the moments response to a 1% increase in the parameters guiding the variances of transitory shocks, $\hat{\gamma}_{a,\iota^+}$ and $\hat{\gamma}_{a,\iota^-}$, respectively. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panels B and D display the simulated positive and negative first autocovariance, respectively, resulting from increasing and decreasing these parameters.

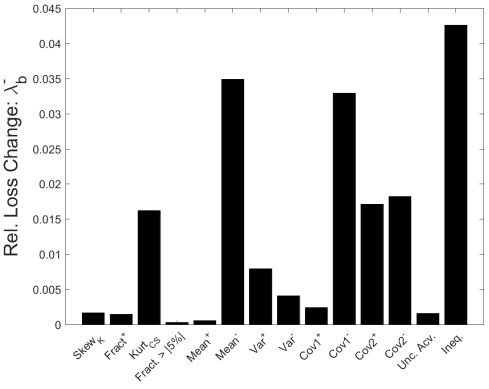
Figure V: Variances of transitory shocks



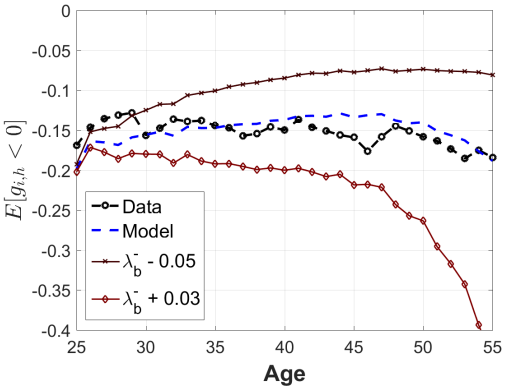
(A) Sensitivity



(B) Simulation



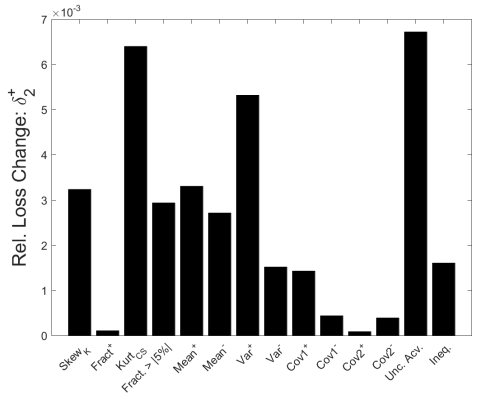
(C) Sensitivity



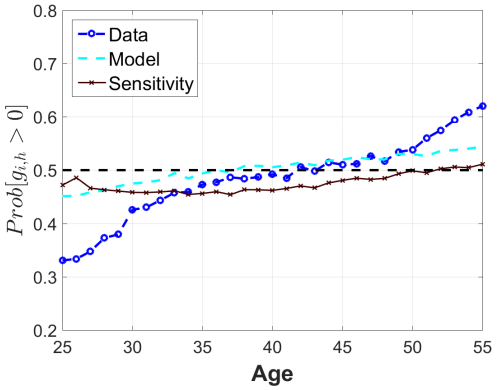
(D) Simulation

Panels A and C display the moments response to a 1% increase in the parameters guiding the means of shocks, $\hat{\lambda}_a^+$ and $\hat{\lambda}_b^-$, respectively. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panels B and D display the simulated positive and negative mean, respectively, resulting from increasing and decreasing these parameters.

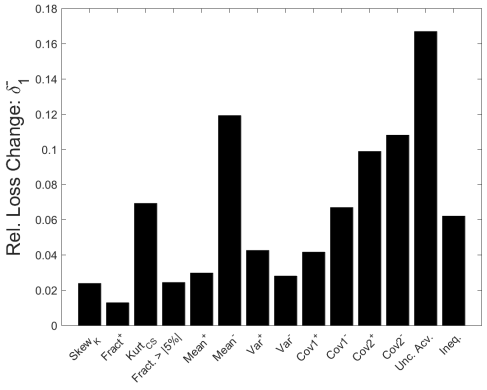
Figure VI: Means of shocks



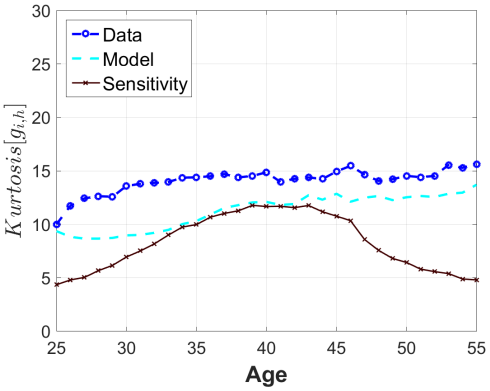
(A) Sensitivity



(B) Simulation



(C) Sensitivity

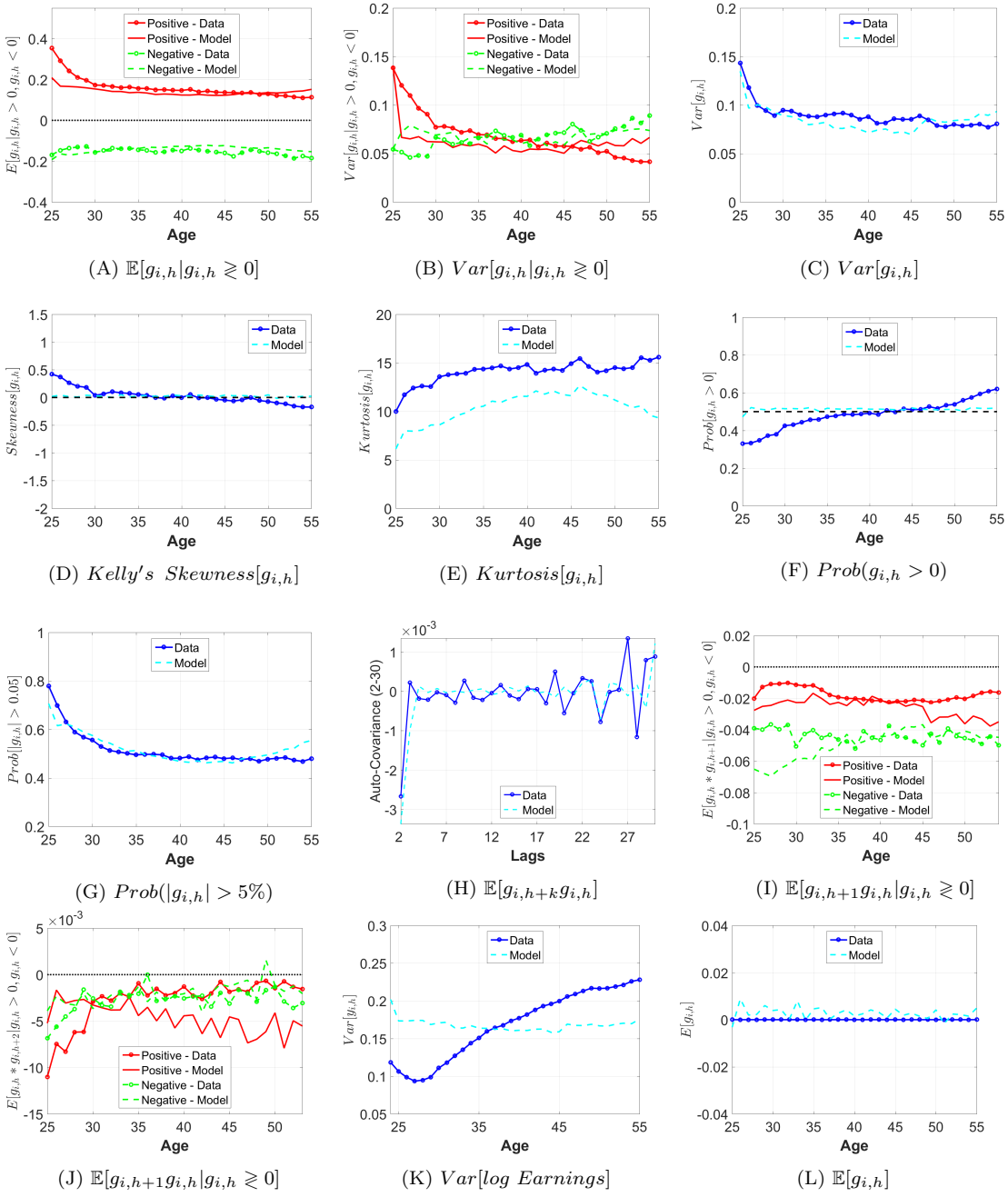


(D) Simulation

Panels A and C display the moments response to a 1% increase in the parameters guiding the sampling probabilities of shocks, $\hat{\delta}_2^+$ and $\hat{\delta}_1^-$, respectively. The order of the moments is: Kelly's skewness, fraction of positive earnings growth, kurtosis, the fraction of earnings growth larger than 0.05, mean positive earnings growth, mean negative earnings growth, variance of positive earnings growth, variance of negative earnings growth, first covariance of positive earnings growth, first covariance of negative earnings growth, second covariance of positive earnings growth, second covariance of negative earnings growth, the unconditional autocovariance, and the variance of log earnings. Panels B and D display selected parameters guiding the probability of positive and negative shocks and the corresponding simulated fraction of positive innovations and kurtosis.

Figure VII: Sampling probabilities

B Model Moments for Alternative Models



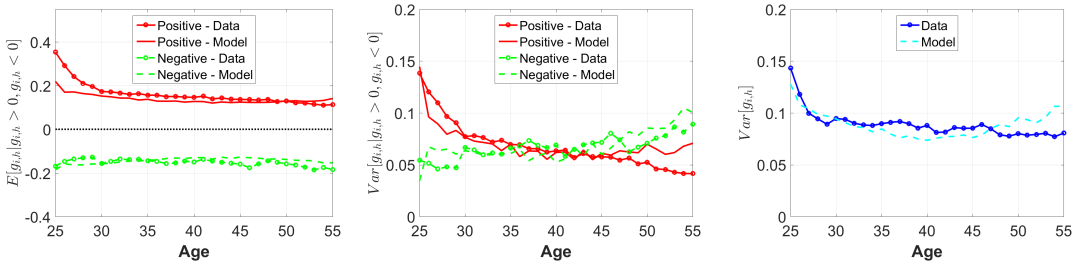
(A) $\mathbb{E}[g_{i,h}|g_{i,h} \gtrless 0]$ (B) $Var[g_{i,h}|g_{i,h} \gtrless 0]$ (C) $Var[g_{i,h}]$

(D) $Kelly's\ \ Skewness[g_{i,h}]$ (E) $Kurtosis[g_{i,h}]$ (F) $Prob(g_{i,h} > 0)$

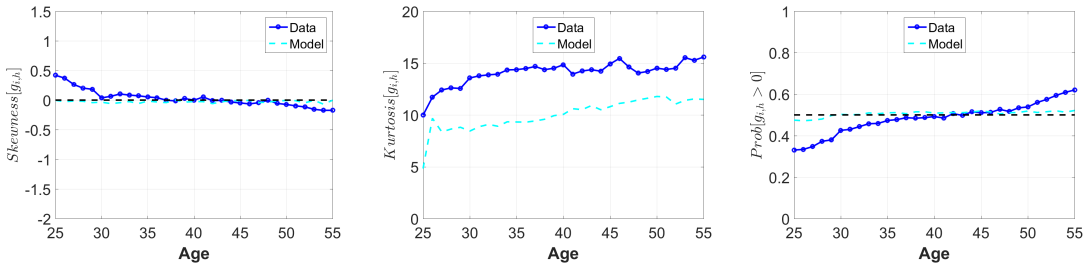
(G) $Prob(|g_{i,h}| > 5\%)$ (H) $\mathbb{E}[g_{i,h+k}g_{i,h}]$ (I) $\mathbb{E}[g_{i,h+1}g_{i,h}|g_{i,h} \gtrless 0]$

(J) $\mathbb{E}[g_{i,h+1}g_{i,h}|g_{i,h} \gtrless 0]$ (K) $Var[log\ Earnings]$ (L) $\mathbb{E}[g_{i,h}]$

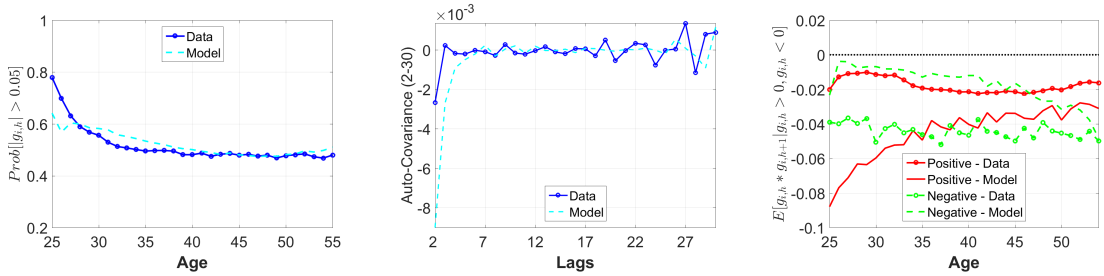
Figure VIII: Model Fit - Column (2) of Table 1 in the Paper



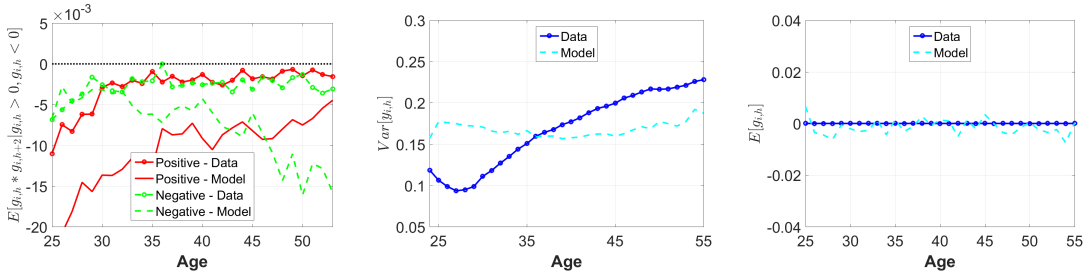
(A) $\mathbb{E}[g_{i,h}|g_{i,h} \gtrless 0]$ (B) $Var[g_{i,h}|g_{i,h} \gtrless 0]$ (C) $Var[g_{i,h}]$



(D) $Kelly's\ \ Skewness[g_{i,h}]$ (E) $Kurtosis[g_{i,h}]$ (F) $Prob(g_{i,h} > 0)$



(G) $Prob(|g_{i,h}| > 5\%)$ (H) $\mathbb{E}[g_{i,h+k}g_{i,h}]$ (I) $\mathbb{E}[g_{i,h+1}g_{i,h}|g_{i,h} \gtrless 0]$



(J) $\mathbb{E}[g_{i,h+2}g_{i,h}|g_{i,h} \gtrless 0]$ (K) $Var[log\ Earnings]$ (L) $\mathbb{E}[g_{i,h}]$

Figure IX: Model Fit - Column (3) of Table 1 in the Paper